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SHAPLEY–SHUBIK POWER INDEX DISTRIBUTION AS A MEASURE FOR DETERMINING THE VALUE OF CONTROL RIGHTS. A THEORETICAL APPROACH

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Abstract: The valuation of control power is a very interesting part of contemporary financial studies. Scholars exploring this field of economic science use a wide scope of concepts and methods. This paper focuses on theoretical aspects of how the sharehoder structure can affect the price of shares.

Key words: corporate rights, control rights valuation, Shapley–Shubik Power Index, publicly traded companies

Theoretically any share gives its propriator two kinds of rights: financial rights and corporate rights. The first group emcompasses: the right to dividend, the right to a part of the resident value in a case of liquidation, the right to sell a share and get an acceptable price for it and so on. The second group consist of quite different privileges: the right to call and take part in shareholders' meetings, right to elect board members, the right to amend a corporate charter, the right to decide about key aspects of the corporate activity. While all finacial rights are separate (that means any shareholder can exercise them without – generally – cooperating with others and without – generally – harming others), excersising of corporate rights sometimes requires some cooperation of other shareholders or – more precisly – taking into account the corporate rights of others.

All the benefits identified with corporate rights assigned to the controlling person(s) encompass knowledge, prestige, control over expenditures of the company and similar gains the controlling person(s) can obtain. A person or a group of persons can excersise the controlling power completely excluding from it other shareholders (sometimes this group of benefits is called *private benefits* [3]). Moreover, the controlling party can influence financial performance of the company and thus harm financial rights of other shareholders. Corporate rights as anything that gives benefits can be priced. Some scholars investigate dual–class companies valuation [4, 5], others use different methods, for example linked with shark repellents effects on the market value of a company [2]. In this paper I assume that in different ownership structures a single transaction of the same numbers of shares affects different changes in shareholders' corporate rights.

In publicly traded companies the price of a single share reflects both rights transferred from a seller to a purchaser. The finacial rights can be measured by several ratios and indicators (ROE, P/E ratio, P/BV ratio, and others). The corporate rights are more complex…

Let's consider the situation of a single shareholder *A* having 10 of 100 shares (that means 10% of voting rights) of a company. In the situation of the lack specific charter regulations he has the right to 10% of the whole dividend, the right to 10% of the resident value and the right to 10% of a sum that an acquirer is ready to pay for a company. It doesn't mean, however, he has 10% of controlling power over a company. Actually, he can have 10% of this power, much more of it or much less of it. It depends of the structure of company's ownership [1]. To illustrate the difference of a single shareholder's position in general shareholders' meeting let's introduce the concept of Shapley–Shubik Power Index (hereafter: SSPI). The basic assumptions of SSPI are:

- the decision–making group consists of *n* persons (A, B, C, … n);
- each person can have different number of votes;
- the persons may create coalitions to get a majority necessary to make a decision;

- the coalition that doesn't have enough votes to make a decision is called the losing coalition; the coalition that have enough votes to make a decision is called the winning coalition;
- coalitions are created by joining another and another person; the coalition {A,B} is different than the coalition {B,A};
- each coalition encompassing all voters has the same probability of existing;
- person who by joining to the losing coalition turn it into the winning coalition is called the pivot voter (P);
- there are *n*! possible joining sequences;
- the Shapley–Shubik Power Index of person *i* is calculated by dividing the number of coalitions in which person *i* is the pivot voter by the number of possible coalitions encompassing all voters (n!).

Probably the most illustrative example of SSPI is a 3–persons decision–making group in which members (A, B, and C) have respectively: 49%, 48% and 3% of votes, and in which a decision needs more than 50% of all votes to be made (all important informations concerning the group can be presented as: {51;49,48,3}). The distribution of voting rights in the described group is deeply unequal, but all members have the same power in a decision–making process: none of them can make a decision alone and must convince at least one of two others. It means any 2–person coalition is a winning coalition and it leads us to a conclusion that all group members have the same possibility to be a pivot voter and thus the same power to influence the decision.

As we can see the shareholder's position in a shareholders meeting depends not only on the number of shares he possesses, but also on the shareholder structure. The same 3% of shares held by shareholder C gives no power to decide if any other shareholder possesses 51% of votes.

Generally, by selling a share a shareholder transfers both rights to a buyer. But it's obvious that not in all cases the same part of control power is transferred to a purchaser. For example, in the structure of three shareholders described above {51;49,48,3} a single transaction on 1% of outstanding shares does not change a position of any shareholder no matter which pair of them would be engaged in this transaction. In other ownership structures a similar transaction can change power of not only engaged parties, but also other group members. To see how deeply one transaction between two shareholders can change the power distribution among all owners consider a 10–member ownership structure company with equal participation (10 shares) of all shareholders: {51; 10, 10, 10, 10, 10, 10, 10, 10, 10, 10} and other 10–member ownership structures.

In the first structure of {51; 10, 10, 10, 10, 10, 10, 10, 10, 10, 10} a single transaction concerning one share will change SSPI of both engaged parties: position of a buyer (say shareholder A) substantially rises; the position of a seller (say: shareholder J) will decrease. All other shareholders will keep initial power. It's illustrated in table 1.

Table 1 – Shapley–Shubik Power Index distribution in equal shares possesion {51; 10, 10, 10, 10, 10, 10, 10, 10, 10, 10} and after a single transaction concerning one share

Shareholder	SSPI (basic)	SSPI after selling one share from J to A
A	10%	15,56%
B	10%	10%
C	10%	10%
D	10%	10%
E	10%	10%
F	10%	10%
G	10%	10%
H	10%	10%
I	10%	10%
J	10%	4,44%

As we can see in the described situation by buing 1% of voting rights shareholder A bought over 5% of power and shareholder J by selling one share resigned from over 5% of control power. Both shareholders engaged in this transaction changed their control power in total by 11,11% (5,55%+5,55%) of overall control over a company. Shareholders not engaged in this transaction saved their control posisions unchanged.

In more complex structures of ownership a single transaction can change control power of even non–engaged shareholders. Thus, we can calculate total change of control power of engaged parties and overall change separately. Let's compare two ownership structures with the same number of sareholders (10) and total number of shares (100), and the same majority rule (51 votes). The structures are: {51; 25, 20, 15, 10, 5, 5, 5, 5, 5, 5} and {51; 40, 25, 15, 10, 5, 1, 1, 1, 1, 1}.

In two tables below (2 and 3) in „the cross sell" defined by a single seller and buyer there are two numbers: the top one describes the total change of control power of both engaged parties of a transaction; the bottom one – the total change of all shareholders' power.

Table 2 – Shapley–Shubik Power Index distribution changes after a single share transaction of: {51; 25, 20, 15, 10, 5, 5, 5, 5, 5, 5}

		Buyer									
		A	B	C	D	E	F	G	H	I	J
Seller	A	—	7,74 8,25	6,75 7,78	6,55 8,75	6,15 7,46	6,15 7,46	6,15 7,46	6,15 7,46	6,15 7,46	6,15 7,46
	B	7,74 8,25	—	5,76 6,35	5,95 7,94	5,55 6,75	5,55 6,75	5,55 6,75	5,55 6,75	5,55 6,75	5,55 6,75
	C	6,75 7,78	5,76 6,35	—	6,15 7,94	5,36 6,35	5,36 6,35	5,36 6,35	5,36 6,35	5,36 6,35	5,36 6,35
	D	5,95 7,94	5,95 7,74	6,15 7,94	—	4,36 5,95	4,36 5,95	4,36 5,95	4,36 5,95	4,36 5,95	4,36 5,95
	E	6,15 7,46	5,55 6,75	5,36 6,35	4,36 5,95	—	4,68 4,68	4,68 4,68	4,68 4,68	4,68 4,68	4,68 4,68
	F	6,15 7,46	5,55 6,75	5,36 6,35	4,36 5,95	4,68 4,68	—	4,68 4,68	4,68 4,68	4,68 4,68	4,68 4,68
	G	6,15 7,46	5,55 6,75	5,36 6,35	4,36 5,95	4,68 4,68	4,68 4,68	—	4,68 4,68	4,68 4,68	4,68 4,68
	H	6,15 7,46	5,55 6,75	5,36 6,35	4,36 5,95	4,68 4,68	4,68 4,68	4,68 4,68	—	4,68 4,68	4,68 4,68
	I	6,15 7,46	5,55 6,75	5,36 6,35	4,36 5,95	4,68 4,68	4,68 4,68	4,68 4,68	4,68 4,68	—	4,68 4,68
	J	6,15 7,46	5,55 6,75	5,36 6,35	4,36 5,95	4,68 4,68	4,68 4,68	4,68 4,68	4,68 4,68	4,68 4,68	—

If we compare changes that a single–share transaction can cause in different ownership structures, we can see that the more equal the SSPI distribution is, the more control power is transferred in a single transaction. Of course, some specific transactions presented in table 3 cause deeper changes in all overall control power than the corresponding transaction presented in table 2 (e.g. when shareholder D sells a share to any shareholder except A), but in all other situations a one–share transaction in a more equal structure causes deeper changes in shareholders power. Can we expect this difference will be observed in quotations of companies with a different distribution of control rights?

Table 3 – Shapley–Shubik Power Index distribution changes after a single share transaction of: {51; 40, 25, 15, 10, 5, 1, 1, 1, 1, 1}

		Buyer									
		A	B	C	D	E	F	G	H	I	J
Seller	A	—	3,77 9,68	3,77 9,68	0,40 1,67	3,37 10,0	2,97 8,89	2,97 8,89	2,97 8,89	2,97 8,89	2,97 8,89
	B	2,78 4,60	—	0,00 0,00	1,39 4,44	1,39 5,24	1,19 4,52	1,19 4,52	1,19 4,52	1,19 4,52	1,19 4,52
	C	2,78 4,60	0,00 0,00	—	1,39 4,44	1,39 5,24	1,19 4,52	1,19 4,52	1,19 4,52	1,19 4,52	1,19 4,52
	D	1,39 5,24	3,37 10,0	3,37 10,0	—	4,76 10,7	3,17 9,52	3,17 9,52	3,17 9,52	3,17 9,52	3,17 9,52
	E	1,39 4,44	0,40 1,67	0,40 1,67	1,79 4,76	—	0,00 0,00	0,00 0,00	0,00 0,00	0,00 0,00	0,00 0,00
	F	1,59 4,37	0,80 1,59	0,80 1,59	1,79 4,76	0,60 1,98	—	0,60 1,98	0,60 1,98	0,60 1,98	0,60 1,98
	G	1,59 4,37	0,80 1,59	0,80 1,59	1,79 4,76	0,60 1,98	0,60 1,98	—	0,60 1,98	0,60 1,98	0,60 1,98
	H	1,59 4,37	0,80 1,59	0,80 1,59	1,79 4,76	0,60 1,98	0,60 1,98	0,60 1,98	—	0,60 1,98	0,60 1,98
	I	1,59 4,37	0,80 1,59	0,80 1,59	1,79 4,76	0,60 1,98	0,60 1,98	0,60 1,98	0,60 1,98	—	0,60 1,98
	J	1,59 4,37	0,80 1,59	0,80 1,59	1,79 4,76	0,60 1,98	0,60 1,98	0,60 1,98	0,60 1,98	0,60 1,98	—

Of course we can expect so, but some constraints limit the ability to effectively measure these differences. Some regulations concerning stock exchange rules (e.g. tender offer restrictions or charter limitations of turnover) as well as charter regulations on board members election can affect the shareholder's ability to sell real control assigned to a share.

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