EVOLUTION OF ELECTROMAGNETIC WAVES OF A POINT SOURCE IN THE SCHWARZSCHILD FIELD

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The problem of electromagnetic radiation of a source in the gravity field of a nearby black hole is considered. A complete class of exact axial-symmetric solutions of the eikonal equations is found. The components of the field tensor are obtained in the large frequency approximation. An expression is found for the strength of the observed radiation as a function of the distance between the black hole and the source, and of the angle between the directions black hole—observer and black hole—source. It is shown that the system consisting of a black hole and a normal star in rotation around the common center of gravity must look like an object of variable brightness.

It is known that a strong gravitational field acting in the neighborhood of a black hole, causes deviation of the light rays, which can influence the angular distribution of the intensity of the radiation of a nearby star. It was shown in [1] that on the focal line, the intensity of light goes up due to the focusing properties of the gravitational field. It follows from this result that a system consisting of a black hole and a normal star, in rotation around their common center of masses, must be observed as an object of variable brightness, even if the normal star is calm.

In order to find the numerical expression of this effect, one has to obtain the exact solution of the system of Maxwell equations in the gravitational field. However, solving this system of equations in a Schwarzschild field by means of expanding functions in a series in powers of r^{-1} is complicated by the fact that the gravitational field near the event horizon surface cannot be considered as weak. The electromagnetic pulses in this area of space suffer considerable perturbations, which are conserved even after the pulses have gone a large distance from the black hole. In spite of this difficulty, a number of interesting results

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were obtained relative to the state of the electromagnetic field near a black hole. In [2, 3] it was shown, for example, that the problem of propagation of the initially flat electromagnetic wave in the Schwarzschild field can be reduced to the problem of solving one second-order differential equation with separating variables. The authors of [4] considered solving the equations obtained by the WKB method in [2], and in the weak field approximation, in [5]. In [6], the Debye potentials were found in the immediate vicinity of the horizon. The existence of several images of an isolated far-away source was discovered in [7, 8]. Unfortunately, the transition from several images observed at $r \gtrsim 2m$ to two images observed when $r \to \infty$, remained unclear. The frequency shift registered by an observer in free fall in the vicinity of a horizon, was studied in [7, 9]. The wave field was found to be a superposition of fields of different frequency. However, no qualitative explanation was given to this interesting result. In [10], a particular solution of the eikonal equation in a Schwarzschild field was found, which allowed them to find the wave transition and reflection coefficients with respect to the surface $r = 3m$.

Since the problem of studying the electromagnetic field in a Schwarzschild field is complicated and cannot be solved exactly in the near future, one has to limit oneself to some frequency approximation. For practical considerations, it is most interesting to find solutions holding in the entire space for large frequencies. These are the solutions we shall be considering below.

Let us start with the Maxwell system of equations in a curved space: $\nabla_\lambda F^{\mu\lambda} = 0$, $\nabla_\lambda F_{\mu\nu} + \nabla_\mu F_{\nu\lambda} + \nabla_\nu F_{\lambda\mu} = 0$. We can eliminate the dependence of $F^{\mu\nu}$ from t and φ by the substitutions

$$F^{\mu\nu} = K^{\mu\nu} e^{i\omega(t-\chi)},\ K^{10} = \sin\varphi L^{10},\ K^{20} = \sin\varphi L^{20},\ K^{30} = \cos\varphi L^{30},$$
$$K^{21} = \sin\varphi L^{21},\ K^{31} = \cos\varphi L^{31},\ K^{32} = \cos\varphi L^{32},\ L^{\mu\nu} = L^{\mu\nu}(v,\ \theta). \tag{1}$$

The eikonal $\psi = \omega(t - \chi)$ satisfies the equation $g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi = 0$. In the case of an axial symmetry, we shall choose the system of coordinates in such a way that $\partial_3\psi = 0$. Then the eikonal equation can be satisfied by the substitutions [11]

$$\partial_1\chi = \frac{-\cos\tau}{1 - \dfrac{2m}{r}},\ \partial_2\chi = \frac{r^2 \sin\tau}{\sqrt{r^2 - 2mr}}. \tag{2}$$

In order to find the equation for τ, we shall compute the quantities $\partial_1\partial_2\chi$ and $\partial_2\partial_1\chi$ from Eq. (2) in two different ways, and equate them. We obtain

$$\sin\tau\frac{\partial\tau}{\partial\theta} = \cos\tau\sqrt{r^2 - 2mr}\frac{\partial\tau}{\partial r} + \sin\tau\frac{r - 3m}{\sqrt{r^2 - 2mr}}. \tag{3}$$

In Eq. (3), the unknown function τ depends on r and θ. To solve Eq. (3), we shall consider θ as the unknown function, and r and τ, as its arguments. To do this, we perform the following substitutions in Eq. (3):

$$\frac{\partial\tau}{\partial\theta} = \frac{1}{\partial\theta/\partial\tau},\ \frac{\partial\tau}{\partial r} = -\frac{\partial\theta/\partial r}{\partial\theta/\partial\tau}. \tag{4}$$

This results in a first-order linear differential equation:

$$-\cos\tau\sqrt{r^2 - 2mr}\frac{\partial\theta}{\partial r} + \frac{r - 2m}{\sqrt{r^2 - 2mr}}\sin\tau\frac{\partial\theta}{\partial\tau} = \sin\tau. \tag{5}$$

Let us introduce a new function ξ:

$$\xi = \frac{2r^2 \sin\tau}{\sqrt{r^2 - 2mr}}. \tag{6}$$

Now it is easy to see that the general solution of Eq. (5) has the form

$$\theta = \int_0^{r^{-1}} \frac{dx}{\sqrt{\dfrac{4}{\xi^2} - x^2 + 2mx^3}} + f_1(\xi), \tag{7}$$

where $f_1(\xi)$ is an arbitrary function of its argument. The expressions (2), together with (6) and (7), give the general solution of the eikonal equation in a Schwarzschild field. The authors of [10] argued that because there is a strong light scattering near the surface $r = 3m$, geometrical optics is not applicable near this surface. This conclusion was made, though, only on the basis of an analysis of trajectories close to those winding around the limit cycle. However, for the trajectories far away from those winding around the limit cycle, the scattering is much weaker. Therefore, the conclusion of [10] about the inapplicability of geometrical optics refers to a class of trajectories, rather than to a domain of space. This is confirmed by the above solution, which possesses no singularities at r = 3m.

Let us find the singularities of the wave front for the case of $f_1(\xi) = 0$. We shall find them as the surfaces of constant phase: $d\chi = 0$. Using this condition as well as Eq. (2), we have

$$\sin\tau\sqrt{r^2-2mr}d\theta = \cos\tau dr. \tag{8}$$

To solve Eq. (8), we developed a program for numerical computation on a microcomputer. The results of computations are given on the figures. In order to make the figures more readable, we show only a half of the line $r = r(\theta)$ in the process of its evolution. The results we have obtained can be interpreted as follows. As one approaches the horizon, the time flows slower and, therefore, the world speed of the wave front gets smaller. This results in the fact that the parts of the wave front far away from the Oz-axis, go faster than the part of the front moving along the Oz-axis, which manifests itself as the "winding" of the wave around the black hole. The stages of this winding are shown on Fig. 1. Due to the winding at $r \to \infty$, the wave front surface, after being "processed" by the gravitational field, transforms into a spiral (in the φ = const cross section) (Fig. 2). The "winding" effect allows for a simple explanation of the results of [7, 9]. The wave winds itself around the black hole, each turn corresponding to a new image of the light source. For an observer in free fall, the observed frequency of the light, due to the Doppler effect, depends on the angle between the velocity vector of the observer and the surface χ = const. With each new turn, this angle becomes different. The "winding" effect results in the fact that a distant observer sees a black hole as an independent light source. For a distant observer, there is almost no difference between normal vectors to different convolutions of the spiral at the observation point. Different images of the light source, observed at infinity, become practically one image. In principle, however, they are different images. The statement of [7] about the existence of several images of the same source is, therefore, valid not only near the horizon surface, but in the whole space, and at infinity one must be able to observe a number of images of the same source, rather than only two images.

Let us consider the angular distribution of the radiation energy. Using the Maxwell system of equations, one can obtain in the limit $\omega \to \infty$ the following relations for $L^{\mu\nu}$:

$$L^{10} = -L^{21}\partial_2\chi,\ L^{20} = L^{21}\partial_1\chi,\ L^{31} = \left(1-\frac{2m}{r}\right)^2\partial_1\chi L^{30},\ L^{32} = \frac{r-2m}{r^3}\partial_2\chi L^{30}. \tag{9}$$

The equations for L^{21} and L^{30} cannot be written in this approximation due to the validity of the eikonal equation. Therefore, in order to find the equations for L^{21} and L^{30}, one has to consider the next approximation of the Maxwell equations in ω^{-1}. The result is

$$\begin{gathered}2\partial_2L^{21}\partial_2\chi + 2(r^2-2mr)\partial_1\chi\partial_1L^{21} + L^{21}\Big[\partial_2\partial_2\chi + \frac{\cos\theta}{\sin\theta}\partial_2\chi + \\ + (r^2-2mr)\partial_1\partial_1\chi + (4r-6m)\partial_1\chi\Big] = 0,\end{gathered} \tag{10}$$

$$\begin{gathered}2\partial_2L^{30}\partial_2\chi + 2(r^2-2mr)\partial_1\chi\partial_1L^{30} + L^{30}\Big[\partial_2\partial_2\chi + 3\frac{\cos\theta}{\sin\theta}\partial_2\chi + \\ + (r^2-2mr)\partial_1\partial_1\chi + (4r-2m)\partial_1\chi\Big] = 0.\end{gathered} \tag{11}$$

It is easy to see that the general solutions of Eqs. (10) and (11) are

$$L^{21} = \sqrt{\frac{\partial_1\partial_2\chi}{r^2\sin\theta}}f_2(\partial_2\chi),\ L^{30} = \frac{1}{(r-2m)\sin\theta}\sqrt{\frac{\partial_1\partial_2\chi}{\sin\theta}}f_3(\partial_2\chi), \tag{12}$$

where $f_2(\partial_2\chi)$ and $f_3(\partial_2\chi)$ are arbitrary functions of their arguments. Their choice is equivalent to choosing initial conditions for the problem. It follows from the results of [12] that a singularity of a light pulse at some point of the wave front — say, a discontinuity of intensity — manifests itself as a singularity of at least one of the functions $f_2(\partial_2\chi)$ or $f_3(\partial_2\chi)$. On the other hand, if one chooses these functions in such a way that they have a singularity at some value of the argument, this singularity will be present at other points of space for the same value of the argument as well. The condition $\partial_2\chi$ = const is, therefore, a condition

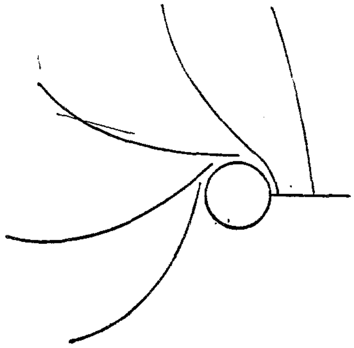

Fig. 1

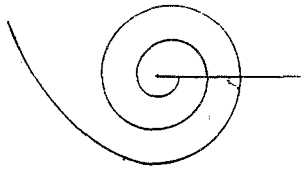

Fig. 2

of propagation in space of some singularities of the electromagnetic pulse (in particular, discontinuities of brightness, or rays).

If the function $f_1(\xi)$ is known, Eq. (7) at each point of space can be considered as an equation with respect to ξ. If there is a light source at some point of space, then, since $\xi = 2\partial_2\chi$, this point belongs to many rays, i.e., Eq. (7) must be valid for a number of ξ at this point. One can use this fact to make a choice of $f_1(\xi)$. For example, if the light source is located at $\theta = 0$ at the distance a from the origin, then one can substitute the values of r and θ into Eq. (7) and find $f_1(\xi)$:

$$f_1(\xi) = -\int_a^{a^{-1}} \frac{dx}{\sqrt{4\cdot\xi^{-2} - x^2 + 2mx^3}}\,. \tag{13}$$

Substituting Eq. (13) into Eq. (7), we find then

$$\theta = \int_{a^{-1}}^{r^{-1}} [4 - \xi^{-2} - x^2 + 2mx^3]^{-1/2}\,dx. \tag{14}$$

In particular, if the light source is located at infinity, $f_1(\xi) = 0$. Introducing in the neighborhood of the point $\theta = 0$, $z = a$ a local pseudo-Cartesian system of coordinates, and changing coordinate components into physical ones, we can easily see that the choice of

$$f_2(\partial_2\chi) = \left(\frac{1}{a} - \frac{2m}{a^2}\right)\left[1 - \left(1 - \frac{2m}{a}\right)\frac{(\partial_2\chi)^2}{a^2}\right]^{1/4},$$
$$f_3(\partial_2\chi) = \frac{\dfrac{1}{a} - \dfrac{2m}{a^2}}{\left[1 - \left(1 - \dfrac{2m}{a}\right)\dfrac{(\partial_2\chi)^2}{a^2}\right]^{1/4}} \tag{15}$$

in Eq. (12) corresponds to a dipole radiation of a charge oscillating in the direction orthogonal to the 0z-axis. The radiating surface of a star can be considered as a system of chaotically oriented dipole sources. For a charge oscillating parallel to the oz-axis, we find in a similar fashion:

$$F^{\mu\nu} = K^{\mu\nu}(r,\ \theta)\cdot e^{i\omega(t-\chi)},\ K^{01} = K^{02} = K^{03} = 0,$$
$$K^{10} = -K^{21}\partial_2\chi,\quad K^{20} = K^{21}\partial_1\chi,$$
$$K^{21} = (\partial_1\partial_2\chi)^{1/2}\cdot r^{-1}\sin^{-1/2}\theta\cdot f_4(\partial_2\chi). \tag{16}$$

Choosing

$$f_4(\partial_2\chi) = \frac{-\left(1 - \dfrac{2m}{a}\right)^{3/2}\partial_2\chi}{a^2\left[1 - \left(1 - \dfrac{2m}{a}\right)\dfrac{(\partial_2\chi)^2}{a^2}\right]^{1/4}}, \tag{17}$$

this solution will describe dipole radiation. Taking into consideration Eqs. (9), (12), and (15)-(17), averaging over all possible directions of oscillation, and changing coordinate components of $F^{\mu\nu}$ into physical ones, we can find the electromagnetic energy density:

$$W = I_0 \frac{\left(1 - \frac{2m}{a}\right)^2 \partial_1\partial_2\chi \sin^2(\omega t - \omega\chi)}{\left(1 - \frac{2m}{r}\right) a^2 \sin\theta \left[1 - \left(1 - \frac{2m}{a}\right)\frac{(\partial_2\chi)^2}{a^2}\right]^{1/2}}. \quad (18)$$

The expression (18) together with Eqs. (2), (6), and (14) gives an implicit exact solution of the problem of finding the intensity of electromagnetic radiation of a source in the vicinity of a black hole. To use these results in practice, we performed a numerical computation of the electromagnetic radiation density. Using the method of least squares, this resulted in the empirical formula:

$$I = \frac{I_0}{r^2 \sin\theta}[5.50564\cdot 10^{-3} + 31.972050\theta + 0.805007\theta^2 - 9.263345\theta^3 + \\ + 7.854750\theta^4 - 7.447314\theta^5 + 3.957080\theta^6 - 1.025960\theta^7 + 0.104708\theta^8], \quad (19)$$

where I_0 is the proper intensity of radiation of the star.

The radius of the orbit was taken to be 100 × m. The precision of this formula is at least 2×10^{-3}. It is equally easy to obtain similar formulas for other values of the radius of the orbit. It follows from Eq. (19) that the intensity of radiation of the system star—black hole depends, in agreement with what was thought before, on the angle between the directions "star—black hole" and "black hole—observer." Because this angle changes periodically when the star and the black hole rotate around their common center of gravity, the observed brightness of radiation changes periodically as well. This means that, in principle, observation of cosmic objects of variable brightness can be used in finding black holes.

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