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DYNAMIC SYSTEMS WITH SMALL PARAMETER

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In this paper using reflecting function [1] and the small parameter method the sufficient conditions of existence of family periodic solutions close to a given solution of multidimensional nonlinear differential systems are obtained (see also [2-9]).

Consider the system

$$\dot{x} = X(t,x), \quad t \in R, \quad x \in R^n, \tag{1}$$

with a continuously differentiable right–hand side and with a general solution $\varphi(t; t_0, x_0)$. For each such system, the *reflecting function* (RF) is defined as $F(t,x) := \varphi(-t; t, x)$. If system (1) is 2ω–periodic with respect to t, and F is its RF, then $F(-\omega, x) = \varphi(\omega; -\omega, x)$ is the mapping of this system over the period $[-\omega, \omega]$.

A function $F(t,x)$ is a reflecting function of system (1) if and only if it is a solution of the system of partial differential equations (called a *basic relation*)

$$\frac{\partial F(t,x)}{\partial t} + \frac{\partial F(t,x)}{\partial x} X(t,x) + X(-t, F(t,x)) = 0$$

with the initial condition $F(0,x) \equiv x$.

Each continuously differentiable function F that satisfies the condition $F(-t, F(t,x)) \equiv F(0,x) \equiv x$, is a RF of the whole class of systems of the form [1]

$$\dot{x} = -\frac{1}{2}\frac{\partial F}{\partial x}(-t, F(t,x))\left(\frac{\partial F(t,x)}{\partial t} - 2S(t,x)\right) - S(-t, F(t,x)), \tag{2}$$

where S is an arbitrary vector function such that the solutions of system (2) are uniquely determined by their initial conditions. Therefore, all systems of the form (1) are split into equivalence classes of the form (2) so that each class is specified by a certain reflecting function referred to as the *RF of the class*. For all systems of one class, the shift operator on the interval $[-\omega, \omega]$ is the same. Therefore, all equivalent 2ω–periodic systems have a common mapping over the period.

Consider the nonlinear differential system depending on parameter ν

$$\dot{x} = f(t, x, \nu), \quad t \in R, \quad x \in D \subset R^n, \tag{3}$$

where f is a continuous ω–periodic vector function for all t, small $|\nu|$, and also continuously differentiable with respect to components of a vector x. Let $x = g_0(t)$ be an ω–periodic solution of the system (3) in which $\nu = 0$.

Theorem I. *Let $F(t)x$ be the RF of the linear system $\dot{x} = \frac{\partial f}{\partial x}(t, g_0(t), 0)x$. If there is no unit among solutions μ_i of the equation $\det\left(F\left(-\frac{\omega}{2}\right) - \mu E\right) = 0$, then system (3) with sufficiently small $|\nu|$ has the unique ω–periodic solution $x = x(t,\nu)$ with an initial point $x(0,\nu)$ close to $g_0(0)$. Besides, $x(t,\nu)$ is a continuous function with respect to (t,ν), and $x(t,0) = g_0(t)$. If, moreover, f is continuously differentiable with respect to ν, then $x(t,\nu)$ is also continuously differentiable.*

Now consider the autonomous differential system depending from parameter ν

$$\dot{x} = f(x,\nu), \quad x \in D \subset R^n, \quad \nu \in R, \tag{4}$$

where f is a continuous vector function with respect to small $|\nu|$ and $x \in D$, also continuously differentiable with respect to components of a vector x. Let $x = \eta(t) \not\equiv \text{const}$ be an ω_0–periodic solution of the system $\dot{x} = f(x, 0)$.

Theorem 2. *Let* $F(t)x$ *be the RF of the linear system* $\dot{x} = \frac{\partial f}{\partial x}(\eta(t), 0)x$. *If among solutions* μ_i *of the equation* $\det\left(F\left(-\frac{\omega_0}{2}\right) - \mu E\right) = 0$ *there is unique simple unit, then system (4) with sufficiently small* $|\nu|$ *has the unique periodic solution* $x = x(t, \nu)$ *close to* $\eta(t)$ *with period* $\omega = \omega(\nu)$ *close to* ω_0. *Moreover,* $x(t,\nu)$ *and* $\omega(\nu)$ *are continuous and* $x(t,0) = \eta(t)$, $\omega(0) = \omega_0$.

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